

Question 10

The displacement $s(t)$ (in meters) of a particle moving along a straight line is given by

$$s(t) = t^3 - 6t^2 + 9t,$$

where t is time in seconds.

- (a) Find the average velocity of the particle over the interval $[1, 4]$.
- (b) Find the instantaneous velocity of the particle at $t = 1$.
- (c) Sketch the displacement function, the secant line over $[1, 4]$, and the tangent line at $t = 1$.

Question 10 - Solution

Given:

$$s(t) = t^3 - 6t^2 + 9t$$

(a) Average velocity on $[1, 4]$:

$$s(1) = 1^3 - 6(1)^2 + 9(1) = 1 - 6 + 9 = 4$$

$$s(4) = 64 - 96 + 36 = 4$$

$$\text{Average velocity} = \frac{4 - 4}{4 - 1} = 0 \text{ m/s}$$

(b) Instantaneous velocity at $t = 1$:

$$s'(t) = 3t^2 - 12t + 9 \Rightarrow s'(1) = 3(1)^2 - 12(1) + 9 = 3 - 12 + 9 = 0 \text{ m/s}$$

(c) **Interpretation:** The average and instantaneous velocities are both zero, indicating the particle returns to its starting displacement at $t = 4$, and is momentarily at rest at $t = 1$.

