

Question 9

The revenue R (in dollars) generated by selling t hundred items is given by the function

$$R(t) = 200t - 3t^2.$$

- (a) Compute the average rate of change of revenue on the interval $[2, 6]$.
- (b) Find the instantaneous rate of change at $t = 2$.
- (c) Sketch the revenue function, the secant line over $[2, 6]$, and the tangent line at $t = 2$.

Question 9 - Solution

Given:

$$R(t) = 200t - 3t^2$$

(a) Average rate of change on $[2, 6]$:

$$R(2) = 200(2) - 3(2)^2 = 400 - 12 = 388$$

$$R(6) = 200(6) - 3(6)^2 = 1200 - 108 = 1092$$

$$\text{Average rate} = \frac{1092 - 388}{6 - 2} = \frac{704}{4} = 176 \text{ dollars per hundred items}$$

(b) Instantaneous rate of change at $t = 2$:

$$R'(t) = 200 - 6t \Rightarrow R'(2) = 200 - 12 = 188$$

(c) **Interpretation:** At $t = 2$, revenue is increasing faster than the average over the interval $[2, 6]$. The tangent line shows this steeper slope.

