

Question 8

The population P (in thousands) of a bacteria culture after t hours is modeled by

$$P(t) = \frac{100t}{1 + 0.5t}.$$

- (a) Compute the average rate of change of the population on the interval $[1, 3]$.
- (b) Find the instantaneous rate of change at $t = 1$.
- (c) Sketch the population function, the secant line over $[1, 3]$, and the tangent line at $t = 1$.

Question 8 - Solution

Given:

$$P(t) = \frac{100t}{1 + 0.5t}$$

(a) Average rate of change on $[1, 3]$:

$$P(1) = \frac{100(1)}{1 + 0.5(1)} = \frac{100}{1.5} \approx 66.67$$

$$P(3) = \frac{100(3)}{1 + 0.5(3)} = \frac{300}{2.5} = 120$$

$$\text{Average rate} = \frac{120 - 66.67}{3 - 1} = \frac{53.33}{2} = 26.67 \text{ thousand/hour}$$

(b) Instantaneous rate of change at $t = 1$:

Use the quotient rule:

$$P'(t) = \frac{(1 + 0.5t)(100) - 100t(0.5)}{(1 + 0.5t)^2} = \frac{100 + 50t - 50t}{(1 + 0.5t)^2} = \frac{100}{(1 + 0.5t)^2}$$

$$P'(1) = \frac{100}{(1 + 0.5)^2} = \frac{100}{2.25} \approx 44.44 \text{ thousand/hour}$$

(c) **Interpretation:** The population is increasing more rapidly at $t = 1$ than the average over the interval. The tangent line shows the instantaneous trend, and the secant line reflects the average change.

