

Question 7

The temperature T (in degrees Celsius) of a cooling object is modeled by

$$T(t) = 25 + 60e^{-0.4t},$$

where t is time in minutes.

- (a) Compute the average rate of change of the temperature over the interval $[1, 4]$.
- (b) Find the instantaneous rate of change of the temperature at $t = 1$.
- (c) Sketch the temperature function, the secant line over $[1, 4]$, and the tangent line at $t = 1$.

Question 7 - Solution

Given:

$$T(t) = 25 + 60e^{-0.4t}$$

(a) Average rate of change on $[1, 4]$:

$$T(1) = 25 + 60e^{-0.4} \approx 25 + 60(0.6703) \approx 65.22$$

$$T(4) = 25 + 60e^{-1.6} \approx 25 + 60(0.2019) \approx 37.11$$

$$\text{Average rate} = \frac{37.11 - 65.22}{4 - 1} \approx \frac{-28.11}{3} \approx -9.37 \text{ }^{\circ}\text{C/min}$$

(b) Instantaneous rate of change at $t = 1$:

$$T'(t) = 60(-0.4)e^{-0.4t} = -24e^{-0.4t}$$

$$T'(1) = -24e^{-0.4} \approx -24(0.6703) \approx -16.09 \text{ }^{\circ}\text{C/min}$$

(c) **Interpretation:** At $t = 1$, the object is cooling at a faster rate than the average cooling over the interval $[1, 4]$, consistent with exponential decay.

