

Question 6

The height h (in meters) of a rocket launched vertically is given by the function

$$h(t) = 100 + 40t - 5t^2,$$

where t is time in seconds after launch.

- (a) Compute the average velocity of the rocket over the time interval $[2, 4]$.
- (b) Find the instantaneous velocity at $t = 2$.
- (c) Sketch the height function, the secant line over $[2, 4]$, and the tangent line at $t = 2$.

Question 6 - Solution

Given:

$$h(t) = 100 + 40t - 5t^2$$

(a) Average velocity on $[2, 4]$:

$$h(2) = 100 + 80 - 20 = 160$$

$$h(4) = 100 + 160 - 80 = 180$$

$$\text{Average velocity} = \frac{180 - 160}{4 - 2} = \frac{20}{2} = 10 \text{ m/s}$$

(b) Instantaneous velocity at $t = 2$:

$$h'(t) = 40 - 10t \Rightarrow h'(2) = 40 - 20 = 20 \text{ m/s}$$

(c) **Interpretation:** At $t = 2$, the rocket is climbing at a rate of 20 m/s, which is faster than its average velocity between $t = 2$ and $t = 4$.

