

Question 5

The position of a particle moving along a straight path is given by the function

$$s(t) = t^3 - 6t^2 + 9t,$$

where $s(t)$ is in meters and t is in seconds.

- (a) Compute the average velocity of the particle on the interval $[1, 3]$.
- (b) Find the instantaneous velocity at $t = 1$.
- (c) Sketch the position function and include both the secant line over $[1, 3]$ and the tangent line at $t = 1$.

Question 5 - Solution

Given:

$$s(t) = t^3 - 6t^2 + 9t$$

(a) Average velocity on $[1, 3]$:

$$s(1) = 1^3 - 6(1)^2 + 9(1) = 1 - 6 + 9 = 4$$

$$s(3) = 27 - 54 + 27 = 0$$

$$\text{Average velocity} = \frac{0 - 4}{3 - 1} = \frac{-4}{2} = -2 \text{ m/s}$$

(b) Instantaneous velocity at $t = 1$:

$$s'(t) = 3t^2 - 12t + 9$$

$$s'(1) = 3(1)^2 - 12(1) + 9 = 3 - 12 + 9 = 0 \text{ m/s}$$

(c) **Interpretation:** The particle is momentarily at rest at $t = 1$ (instantaneous velocity is 0), even though its average velocity between $t = 1$ and $t = 3$ is negative, indicating motion reversal.

