

Question 4

A ball is thrown vertically upward from the ground with an initial velocity of $v_0 = 20$ m/s. Its height at time t seconds is modeled by the function

$$h(t) = 20t - 4.9t^2.$$

- (a) Compute the average velocity over the interval $[1, 2]$.
- (b) Find the instantaneous velocity at $t = 1$.
- (c) Sketch the height function and both the secant and tangent lines involved.

Question 4 - Solution

Given:

$$h(t) = 20t - 4.9t^2$$

(a) Average velocity on $[1, 2]$:

$$h(1) = 20(1) - 4.9(1)^2 = 20 - 4.9 = 15.1$$

$$h(2) = 20(2) - 4.9(4) = 40 - 19.6 = 20.4$$

$$\text{Average velocity} = \frac{20.4 - 15.1}{2 - 1} = 5.3 \text{ m/s}$$

(b) Instantaneous velocity at $t = 1$:

$$h'(t) = 20 - 9.8t \Rightarrow h'(1) = 20 - 9.8 = 10.2 \text{ m/s}$$

(c) **Interpretation:** The ball's velocity at exactly $t = 1$ is greater than the average velocity over the interval, indicating it is still on the way up. Below is the sketch of the graph.

