

Question 3

A population of bacteria in a lab experiment is modeled by the function

$$P(t) = 200e^{0.3t},$$

where t is the time in hours since the start of the experiment and $P(t)$ is the number of bacteria.

- (a) Estimate the average rate of growth of the population from $t = 2$ to $t = 5$.
- (b) Find the instantaneous rate of growth at $t = 2$.
- (c) Sketch the population function along with the secant and tangent lines.

Question 3 - Solution

We are given:

$$P(t) = 200e^{0.3t}$$

(a) **Average rate of change on** $[2, 5]$:

$$P(2) = 200e^{0.6} \approx 200(1.8221) = 364.42$$

$$P(5) = 200e^{1.5} \approx 200(4.4817) = 896.34$$

$$\text{Average rate} = \frac{896.34 - 364.42}{5 - 2} \approx \frac{531.92}{3} \approx 177.31 \text{ bacteria/hr}$$

(b) **Instantaneous rate at** $t = 2$:

$$P'(t) = 200 \cdot 0.3e^{0.3t} = 60e^{0.3t}$$

$$P'(2) = 60e^{0.6} \approx 60(1.8221) = 109.33 \text{ bacteria/hr}$$

(c) **Interpretation:** The population grows faster over the longer interval than it does exactly at $t = 2$, illustrating exponential acceleration.

