

Question 1

Let $f(x) = \sqrt{x+4}$. Find the slope of the secant line through the points where $x = 0$ and $x = 5$, and compare it with the slope of the tangent line at $x = 2$. Then, write the equation of the tangent line at $x = 2$.

Question 1 - Solution

We are given:

$$f(x) = \sqrt{x+4}$$

Step 1: Slope of secant line from $x = 0$ to $x = 5$.

$$f(0) = \sqrt{4} = 2, \quad f(5) = \sqrt{9} = 3$$

$$\text{Secant slope} = \frac{3-2}{5-0} = \frac{1}{5}$$

Step 2: Derivative of $f(x)$.

$$f'(x) = \frac{1}{2\sqrt{x+4}}$$

$$f'(2) = \frac{1}{2\sqrt{6}} \approx 0.204$$

Step 3: Point at $x = 2$.

$$f(2) = \sqrt{6}$$

Step 4: Tangent line at $x = 2$:

$$y - \sqrt{6} = \frac{1}{2\sqrt{6}}(x - 2)$$

Comparison: The slope of the tangent line at $x = 2$ is slightly greater than the secant slope over the interval, showing that the graph is concave down.

