

Question 9

Solve for all real solutions:

$$\log(x^2 + 1) = 1 + \log(x - 2)$$

Instructions: Solve algebraically and check for extraneous solutions.

Question 9 - Solution

We are given:

$$\log(x^2 + 1) = 1 + \log(x - 2)$$

Step 1: Use properties of logarithms.

Subtract $\log(x - 2)$ from both sides:

$$\log(x^2 + 1) - \log(x - 2) = 1$$

Apply the quotient rule:

$$\log\left(\frac{x^2 + 1}{x - 2}\right) = 1$$

Step 2: Convert to exponential form.

Recall that $\log_b(A) = C \Rightarrow A = b^C$. Since no base is written, it is base 10:

$$\frac{x^2 + 1}{x - 2} = 10^1 = 10$$

$$x^2 + 1 = 10(x - 2) \Rightarrow x^2 + 1 = 10x - 20 \Rightarrow x^2 - 10x + 21 = 0 \Rightarrow (x - 3)(x - 7) = 0 \Rightarrow x = 3 \text{ or } x = 7$$

Step 3: Domain check.

We need: $x - 2 > 0 \Rightarrow x > 2$

Both $x = 3$ and $x = 7$ satisfy the domain.

Final Answer:

$x = 3 \quad \text{or} \quad x = 7$
