

Question 7

Solve for all real solutions:

$$\log_2(x + 3) + \log_2(x - 1) = 3$$

Instructions: Solve algebraically and check for extraneous solutions.

Question 7 - Solution

We are given:

$$\log_2(x+3) + \log_2(x-1) = 3$$

Step 1: Combine the logs using the product rule:

$$\log_2[(x+3)(x-1)] = 3$$

Step 2: Convert from logarithmic to exponential form:

$$(x+3)(x-1) = 2^3 = 8 \Rightarrow x^2 + 2x - 3 = 8 \Rightarrow x^2 + 2x - 11 = 0$$

Step 3: Solve the quadratic:

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-11)}}{2(1)} = \frac{-2 \pm \sqrt{4+44}}{2} = \frac{-2 \pm \sqrt{48}}{2} = \frac{-2 \pm 4\sqrt{3}}{2} = \boxed{-1 \pm 2\sqrt{3}}$$

Step 4: Domain check:

We need:

$$x+3 > 0 \Rightarrow x > -3, \quad x-1 > 0 \Rightarrow x > 1 \Rightarrow \text{So } x > 1$$

Now test the two roots: $x = -1+2\sqrt{3} \approx -1+3.464 \approx 2.464 \rightarrow \text{valid}$, $x = -1-2\sqrt{3} \approx -1-3.464 \approx -4.464 \rightarrow \text{invalid}$

Final Answer:

$$\boxed{x = -1 + 2\sqrt{3}}$$