

Question 6

Solve the equation:

$$4e^{2x} - 12e^x + 5 = 0$$

- (a) Solve algebraically for all real solutions.
- (b) Provide both exact and approximate answers.

Question 6 - Solution

(a) **Let** $y = e^x$. Then the equation becomes:

$$4y^2 - 12y + 5 = 0$$

Use the quadratic formula:

$$y = \frac{12 \pm \sqrt{(-12)^2 - 4(4)(5)}}{2(4)} = \frac{12 \pm \sqrt{144 - 80}}{8} = \frac{12 \pm \sqrt{64}}{8} = \frac{12 \pm 8}{8}$$

So:

$$y = \frac{20}{8} = 2.5 \quad \text{or} \quad y = \frac{4}{8} = 0.5$$

Now solve for x :

$$e^x = 2.5 \Rightarrow x = \ln(2.5), \quad e^x = 0.5 \Rightarrow x = \ln(0.5)$$

(b) Final Answers:

Exact:

$$\boxed{x = \ln(2.5) \quad \text{or} \quad x = \ln(0.5)}$$

Approximate:

$$x \approx \ln(2.5) \approx 0.9163, \quad x \approx \ln(0.5) \approx -0.6931$$