

Question 5

Solve the equation:

$$\ln(x - 1) + \ln(x + 1) = \ln(6)$$

- (a) Solve for x algebraically.
- (b) Identify and exclude any extraneous solutions.

Question 5 - Solution

(a) Combine the logarithmic expressions using properties:

$$\ln(x-1) + \ln(x+1) = \ln[(x-1)(x+1)] = \ln(x^2-1)$$

So the equation becomes:

$$\ln(x^2-1) = \ln(6) \Rightarrow x^2-1 = 6 \Rightarrow x^2 = 7 \Rightarrow x = \pm\sqrt{7}$$

(b) **Domain Check:**

We must have:

$$x-1 > 0 \Rightarrow x > 1 \quad \text{and} \quad x+1 > 0 \Rightarrow x > -1 \Rightarrow \text{Combined: } x > 1$$

So $x = \sqrt{7} \approx 2.6458$ is valid, but $x = -\sqrt{7} \approx -2.6458$ is not.

$$\text{Final Answer: } \boxed{x = \sqrt{7}}$$