

Question 3

Solve the following equation:

$$\log_3(x^2 - 4x) = 1 + \log_3(x - 2)$$

- (a) Solve the equation algebraically.
- (b) Identify any extraneous solutions.

Question 3 - Solution

(a) **Combine the logarithms:**

Move the constant:

$$\log_3(x^2 - 4x) - \log_3(x - 2) = 1$$

Apply the quotient rule:

$$\log_3\left(\frac{x^2 - 4x}{x - 2}\right) = 1$$

Factor the numerator:

$$x^2 - 4x = x(x - 4) \Rightarrow \frac{x(x - 4)}{x - 2}$$

So:

$$\log_3\left(\frac{x(x - 4)}{x - 2}\right) = 1 \Rightarrow \frac{x(x - 4)}{x - 2} = 3^1 = 3$$

Multiply both sides:

$$x(x - 4) = 3(x - 2) \Rightarrow x^2 - 4x = 3x - 6 \Rightarrow x^2 - 7x + 6 = 0 \Rightarrow (x - 6)(x - 1) = 0 \Rightarrow x = 6, 1$$

(b) **Check for extraneous solutions:**

Logarithms are only defined for positive arguments.

Check domain: $x^2 - 4x > 0 \Rightarrow x(x - 4) > 0 \Rightarrow x < 0$ or $x > 4$, $x - 2 > 0 \Rightarrow x > 2$

Only values that satisfy both: $x > 4$

So: $x = 6 \rightarrow$ valid, $x = 1 \rightarrow$ not valid (fails both domain conditions)

Final Answer:

$$\boxed{x = 6}$$