

Question 10

Solve the logarithmic equation:

$$\ln(x + 2) + \ln(x - 1) = \ln(3x)$$

- (a) Solve the equation algebraically.
- (b) State the domain and eliminate any extraneous solutions.

Question 10 - Solution**(a) Combine the logarithmic expressions:**

Use the product rule:

$$\ln[(x+2)(x-1)] = \ln(3x) \Rightarrow (x+2)(x-1) = 3x \Rightarrow x^2 + x - 2 = 3x \Rightarrow x^2 - 2x - 2 = 0$$

Use the quadratic formula:

$$x = \frac{2 \pm \sqrt{(-2)^2 + 4(1)(2)}}{2} = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = \boxed{1 \pm \sqrt{3}}$$

(b) Domain restrictions:

We need all logarithms to have positive arguments:

$$x+2 > 0 \Rightarrow x > -2, \quad x-1 > 0 \Rightarrow x > 1, \quad 3x > 0 \Rightarrow x > 0$$

Combining all: $\boxed{x > 1}$ Now test each solution: - $x = 1 + \sqrt{3} \approx 2.73 > 1 \rightarrow$ valid - $x = 1 - \sqrt{3} \approx -0.73 < 1 \rightarrow$ invalid (violates domain)

$$\text{Final answer: } \boxed{x = 1 + \sqrt{3}}$$