

Question 9

A bacteria culture grows according to the function

$$N(t) = N_0 e^{kt}$$

where N_0 is the initial population, k is a constant, and t is the time in hours.

Suppose a culture starts with 500 bacteria and grows to 2000 bacteria in 6 hours.

- (a) Find the growth rate k .
- (b) Use your result to estimate the population after 10 hours.
- (c) After how many hours will the population reach 10,000?

Question 9 - Solution

Growth rate. $2000 = 500e^{6k}$ gives

$$k = \frac{\ln 4}{6} \approx 0.231049 \text{ hr}^{-1}.$$

Population after ten hours.

$$N(10) = 500 \cdot 4^{10/6} \approx 5039.68,$$

so the estimated whole-number population is $\boxed{5040}$.

Time to reach 10000.

$$500e^{kt} = 10000 \quad \implies \quad t = \frac{6 \ln 20}{\ln 4} \approx 12.97 \text{ hours}.$$

Substitution into the exact model verifies both target populations.