

Question 7

Solve the following logarithmic equation:

$$2\log_4(x+1) - \log_4(x-3) = 1$$

- (a) Solve the equation algebraically.
- (b) State any domain restrictions and eliminate extraneous solutions.

Question 7 - Solution**(a) Combine the logarithms:**

Use the power rule:

$$\log_4(x+1)^2 - \log_4(x-3) = \log_4\left(\frac{(x+1)^2}{x-3}\right)$$

So the equation becomes:

$$\log_4\left(\frac{(x+1)^2}{x-3}\right) = 1 \Rightarrow \frac{(x+1)^2}{x-3} = 4^1 = 4$$

Multiply both sides by $x-3$:

$$(x+1)^2 = 4(x-3) \Rightarrow x^2 + 2x + 1 = 4x - 12 \Rightarrow x^2 - 2x + 13 = 0$$

Solve with the quadratic formula:

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(13)}}{2} = \frac{2 \pm \sqrt{4 - 52}}{2} = \frac{2 \pm \sqrt{-48}}{2} = \frac{2 \pm 4i\sqrt{3}}{2} = 1 \pm 2i\sqrt{3}$$

(b) Domain restrictions:

$$x+1 > 0 \Rightarrow x > -1, \quad x-3 > 0 \Rightarrow x > 3$$

So domain is: $x > 3$

The resulting quadratic has no real roots, so the original real logarithmic equation has no solution.

No real solution.