

Question 5

Let $f(x) = \log_5(x - 1) + \log_5(x + 3)$.

- (a) Determine the domain of $f(x)$.
- (b) Simplify the expression using logarithmic properties.
- (c) Solve the equation $\log_5(x - 1) + \log_5(x + 3) = 2$.

Question 5 - Solution**(a) Domain:**

We need both logarithmic expressions to be defined:

$$x - 1 > 0 \Rightarrow x > 1, \quad x + 3 > 0 \Rightarrow x > -3$$

The more restrictive condition is $x > 1$, so:

Domain: $(1, \infty)$

(b) Use the product rule for logarithms:

$$f(x) = \log_5[(x-1)(x+3)] = \log_5(x^2 + 2x - 3)$$

(c) Solve the equation:

$$\log_5(x^2 + 2x - 3) = 2 \Rightarrow x^2 + 2x - 3 = 5^2 = 25 \Rightarrow x^2 + 2x - 28 = 0$$

Use the quadratic formula:

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{(2)^2 + 4 \cdot 28}}{2} = \frac{-2 \pm \sqrt{4 + 112}}{2} = \frac{-2 \pm \sqrt{116}}{2} \\ &= \frac{-2 \pm 2\sqrt{29}}{2} = -1 \pm \sqrt{29} \end{aligned}$$

Check for extraneous solutions:

$$-1 + \sqrt{29} \approx -1 + 5.385 \approx 4.385 \text{ (valid)}, \quad -1 - \sqrt{29} < 0 \text{ (invalid)}$$

Only the positive root satisfies $x > 1$, so:

$x = -1 + \sqrt{29}$