

Question 4

Let $f(x) = \ln(x^2 - 6x + 10)$.

- (a) Determine the domain of $f(x)$.
- (b) Find the exact value of $f(3)$.
- (c) Solve the equation $\ln(x^2 - 6x + 10) = 0$.

Question 4 - Solution**(a) Domain:**

We require the argument of the logarithm to be positive:

$$x^2 - 6x + 10 > 0$$

Consider the quadratic:

$$x^2 - 6x + 10 = (x - 3)^2 + 1$$

Since $(x - 3)^2 \geq 0$, we know:

$$(x - 3)^2 + 1 \geq 1 > 0 \quad \text{for all real } x$$

Domain: $(-\infty, \infty)$

(b) Evaluate $f(3)$:

$$f(3) = \ln(3^2 - 6 \cdot 3 + 10) = \ln(9 - 18 + 10) = \ln(1) = \boxed{0}$$

(c) Solve $\ln(x^2 - 6x + 10) = 0$:

$$\ln(x^2 - 6x + 10) = 0 \Rightarrow x^2 - 6x + 10 = e^0 = 1 \Rightarrow x^2 - 6x + 9 = 0 \Rightarrow (x - 3)^2 = 0 \Rightarrow \boxed{x = 3}$$