

Question 3

Consider the function:

$$f(x) = \log_2(4x^2 - 12x)$$

- (a) Determine the domain of the function.
- (b) Solve the equation $f(x) = 3$.
- (c) State any restrictions or extraneous solutions.

Question 3 - Solution**(a) Domain:**

We must have:

$$4x^2 - 12x > 0 \Rightarrow 4x(x - 3) > 0$$

Using a sign chart or test intervals:

- Critical points: $x = 0$, $x = 3$ - Positive intervals: $x < 0$ or $x > 3$

$$\text{Domain: } \boxed{(-\infty, 0) \cup (3, \infty)}$$

(b) Solve $f(x) = 3$:

$$\log_2(4x^2 - 12x) = 3 \Rightarrow 4x^2 - 12x = 2^3 = 8 \Rightarrow 4x^2 - 12x - 8 = 0$$

Divide by 4:

$$x^2 - 3x - 2 = 0 \Rightarrow x = \frac{3 \pm \sqrt{(-3)^2 + 4 \cdot 2}}{2} = \frac{3 \pm \sqrt{9 + 8}}{2} = \frac{3 \pm \sqrt{17}}{2}$$

$$\boxed{x = \frac{3 \pm \sqrt{17}}{2}}$$

(c) Check for extraneous solutions:

Approximate roots:

$$\sqrt{17} \approx 4.12 \Rightarrow x \approx \frac{3 \pm 4.12}{2} \Rightarrow x_1 \approx 3.56, \quad x_2 \approx -0.56$$

From domain: $x < 0$ or $x > 3$, so both are valid.

$$\boxed{\text{No extraneous solutions}}$$