

Question 1

Solve the equation for x , showing all steps:

$$\log_3(x + 5) + \log_3(x - 2) = 2$$

- (a) Solve the logarithmic equation algebraically.
- (b) State any restrictions on the domain.
- (c) Check for extraneous solutions.

Question 1 - Solution

(a) Combine the logs using log rules:

$$\log_3(x+5) + \log_3(x-2) = \log_3[(x+5)(x-2)] = 2 \Rightarrow \log_3(x^2 + 3x - 10) = 2$$

Rewrite in exponential form:

$$x^2 + 3x - 10 = 3^2 = 9 \Rightarrow x^2 + 3x - 10 = 9 \Rightarrow x^2 + 3x - 19 = 0$$

Solve with quadratic formula:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-19)}}{2(1)} = \frac{-3 \pm \sqrt{9 + 76}}{2} = \frac{-3 \pm \sqrt{85}}{2}$$

(b) **Domain restrictions:**

Since $\log_3(x+5)$ and $\log_3(x-2)$ are defined only if:

$$x+5 > 0 \Rightarrow x > -5, \quad x-2 > 0 \Rightarrow x > 2 \Rightarrow \text{Overall domain: } x > 2$$

(c) **Check solutions:**

$$\sqrt{85} \approx 9.22 \Rightarrow x_1 = \frac{-3 + 9.22}{2} \approx 3.11, \quad x_2 = \frac{-3 - 9.22}{2} \approx -6.11$$

Only $x_1 \approx 3.11$ satisfies $x > 2$. So:

$$\boxed{x = \frac{-3 + \sqrt{85}}{2}} \quad (\text{valid solution})$$