

Question 9

Solve the equation

$$\tan^2(x) - 3\tan(x) + 2 = 0$$

for all $x \in [0, 2\pi)$.

Question 9 - Solution

Treat this as a quadratic in $\tan(x)$. Let $u = \tan(x)$:

$$u^2 - 3u + 2 = 0 \Rightarrow u = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(2)}}{2(1)} = \frac{3 \pm \sqrt{9 - 8}}{2} = \frac{3 \pm 1}{2} \Rightarrow u = 1, 2$$

So:

$$\tan(x) = 1 \quad \text{or} \quad \tan(x) = 2$$

Solve $\tan(x) = 1$:

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

Solve $\tan(x) = 2$:

We must use the inverse tangent:

$$x = \tan^{-1}(2) \approx 1.107 \text{ radians}$$

Tangent is positive in quadrants I and III, so:

$$x = \tan^{-1}(2), \pi + \tan^{-1}(2)$$

Approximate values:

$$x \approx 1.107, 4.249$$

Final Answer:

$x = \frac{\pi}{4}, \frac{5\pi}{4}, \tan^{-1}(2), \pi + \tan^{-1}(2)$