

Question 8

Solve the equation

$$\sin^2(x) - \cos(x) = 0$$

for all $x \in [0, 2\pi]$.

Question 8 - Solution

We start with the identity:

$$\sin^2(x) = 1 - \cos^2(x)$$

Substitute into the equation:

$$1 - \cos^2(x) - \cos(x) = 0 \Rightarrow -\cos^2(x) - \cos(x) + 1 = 0 \Rightarrow \cos^2(x) + \cos(x) - 1 = 0$$

Let $u = \cos(x)$, then:

$$u^2 + u - 1 = 0 \Rightarrow u = \frac{-1 \pm \sqrt{1^2 + 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$$

So:

$$\cos(x) = \frac{-1 + \sqrt{5}}{2} \approx 0.618, \quad \cos(x) = \frac{-1 - \sqrt{5}}{2} \approx -1.618$$

Only $\frac{-1 + \sqrt{5}}{2}$ is in the range $[-1, 1]$

Now solve:

$$\cos(x) = \frac{-1 + \sqrt{5}}{2} \Rightarrow x = \cos^{-1}\left(\frac{-1 + \sqrt{5}}{2}\right)$$

Let's call $\theta = \cos^{-1}\left(\frac{-1 + \sqrt{5}}{2}\right)$

Since cosine is positive in quadrants I and IV, and $\cos(x) > 0$, we get:

$$x = \theta, \quad x = 2\pi - \theta$$

So:

$$x = \cos^{-1}\left(\frac{-1 + \sqrt{5}}{2}\right), \quad x = 2\pi - \cos^{-1}\left(\frac{-1 + \sqrt{5}}{2}\right)$$

These are the exact answers in terms of inverse cosine.