

Question 7

Solve the equation

$$\sin(2x) = \sqrt{3} \cos(x)$$

for all $x \in [0, 2\pi]$.

Question 7 - Solution

Use the identity:

$$\sin(2x) = 2 \sin(x) \cos(x)$$

Substitute into the equation:

$$2 \sin(x) \cos(x) = \sqrt{3} \cos(x)$$

Subtract $\sqrt{3} \cos(x)$ from both sides:

$$2 \sin(x) \cos(x) - \sqrt{3} \cos(x) = 0 \Rightarrow \cos(x) (2 \sin(x) - \sqrt{3}) = 0$$

Now solve each factor:

****Case 1:**** $\cos(x) = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

****Case 2:**** $2 \sin(x) - \sqrt{3} = 0 \Rightarrow \sin(x) = \frac{\sqrt{3}}{2}$

$$\sin(x) = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}$$

Final Solution:

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}$
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