

Question 6

Solve the equation

$$2 \cos^2(x) - 1 = \sin(x)$$

for all $x \in [0, 2\pi]$.

Question 6 - Solution

We begin by using the Pythagorean identity:

$$\cos^2(x) = 1 - \sin^2(x)$$

Substitute into the equation:

$$2(1 - \sin^2(x)) - 1 = \sin(x) \Rightarrow 2 - 2\sin^2(x) - 1 = \sin(x) \Rightarrow -2\sin^2(x) - \sin(x) + 1 = 0$$

Multiply through by -1 :

$$2\sin^2(x) + \sin(x) - 1 = 0$$

Let $u = \sin(x)$. Then:

$$2u^2 + u - 1 = 0 \Rightarrow u = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)} = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}$$

$$u = \frac{1}{2}, \quad u = -1$$

So:

$$\sin(x) = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\sin(x) = -1 \Rightarrow x = \frac{3\pi}{2}$$

Final solution:

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$