

Question 5

Solve the equation

$$\sin(x) + \sqrt{3}\cos(x) = 0$$

for all $x \in [0, 2\pi]$.

Question 5 - Solution

We start by solving:

$$\sin(x) + \sqrt{3}\cos(x) = 0 \Rightarrow \sin(x) = -\sqrt{3}\cos(x)$$

Divide both sides by $\cos(x)$, assuming $\cos(x) \neq 0$:

$$\tan(x) = -\sqrt{3}$$

Now solve:

$$\tan(x) = -\sqrt{3} \Rightarrow x = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$$

But we want values in $[0, 2\pi]$

So find angles with tangent $= -\sqrt{3}$

The reference angle is $\frac{\pi}{3}$, and tangent is negative in quadrants II and IV

$$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}, \quad x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$\boxed{x = \frac{2\pi}{3}, \frac{5\pi}{3}}$$

Also check if $\cos(x) = 0$

If $\cos(x) = 0 \Rightarrow \sin(x) = 0$ also must hold (from original equation)

But: - If $\cos(x) = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ - At $x = \frac{\pi}{2}$: $\sin(x) = 1$, LHS $= 1 + 0 \neq 0$ - At $x = \frac{3\pi}{2}$: $\sin(x) = -1$, LHS $= -1 + 0 \neq 0$

So those are not valid. Final solution:

$$\boxed{x = \frac{2\pi}{3}, \frac{5\pi}{3}}$$