

Question 4

Solve the equation

$$\cos(2x) + \sin(x) = 0$$

for all $x \in [0, 2\pi]$.

Question 4 - Solution**Step 1: Use a double-angle identity for $\cos(2x)$:**

We use:

$$\cos(2x) = 1 - 2\sin^2(x)$$

Substitute into the equation:

$$1 - 2\sin^2(x) + \sin(x) = 0 \Rightarrow -2\sin^2(x) + \sin(x) + 1 = 0$$

Multiply through by -1 to simplify:

$$2\sin^2(x) - \sin(x) - 1 = 0$$

Step 2: Solve the quadratic equation in $\sin(x)$:Let $u = \sin(x)$:

$$2u^2 - u - 1 = 0 \Rightarrow u = \frac{1 \pm \sqrt{1^2 + 4(2)(1)}}{2(2)} = \frac{1 \pm \sqrt{9}}{4} = \frac{1 \pm 3}{4}$$

So:

$$u = 1 \quad \text{or} \quad u = -\frac{1}{2}$$

Now solve $\sin(x) = 1 \Rightarrow x = \frac{\pi}{2}$
And $\sin(x) = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$ **Final answer:**

$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$
--