

Question 3

Solve the equation

$$\sin(x) \cos(x) = \frac{1}{2}$$

for all $x \in [0, 2\pi]$.

Question 3 - Solution

We begin by using a known identity:

$$\sin(2x) = 2 \sin(x) \cos(x) \Rightarrow \sin(x) \cos(x) = \frac{1}{2} \sin(2x)$$

Substitute into the equation:

$$\frac{1}{2} \sin(2x) = \frac{1}{2} \Rightarrow \sin(2x) = 1$$

Now solve:

$$\sin(2x) = 1 \Rightarrow 2x = \frac{\pi}{2} + 2n\pi \quad \text{for integer } n \Rightarrow x = \frac{\pi}{4} + n\pi$$

Find solutions in $[0, 2\pi]$:

$$n = 0 \Rightarrow x = \frac{\pi}{4} \quad n = 1 \Rightarrow x = \frac{5\pi}{4} \quad n = 2 \Rightarrow x = \frac{9\pi}{4} > 2\pi \rightarrow \text{discard}$$

$$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$$