

Question 2

Solve the equation

$$\cos(2x) = 2 \cos^2(x) - 1$$

for all $x \in [0, 2\pi]$, and verify the identity used.

Question 2 - Solution**Step 1: Recognize the identity.**

The right-hand side is a known double-angle identity:

$$\cos(2x) = 2 \cos^2(x) - 1$$

So this equation is always true:

$$\cos(2x) = \cos(2x)$$

Thus, it holds for all x .**But the problem is to solve:**

$$\cos(2x) = 2 \cos^2(x) - 1$$

Since it holds for all x , this is an identity. So the equation is true for all $x \in [0, 2\pi]$

$x \in [0, 2\pi]$

Verification of identity:

Start with RHS:

$$2 \cos^2(x) - 1 = \cos(2x)$$

This is a standard double-angle identity for cosine:

$$\cos(2x) = 2 \cos^2(x) - 1 \Rightarrow \text{Identity verified.}$$