

Question 1

Solve the equation:

$$2 \sin^2(x) - 3 \sin(x) + 1 = 0$$

for all $x \in [0, 2\pi]$.

Question 1 - Solution

We factor the equation:

$$2 \sin^2(x) - 3 \sin(x) + 1 = 0.$$

Looking for factors of $2 \cdot 1 = 2$ that add to -3 , we write:

$$(2 \sin(x) - 1)(\sin(x) - 1) = 0.$$

Setting each factor equal to zero:

$$2 \sin(x) - 1 = 0 \quad \text{or} \quad \sin(x) - 1 = 0.$$

$$\sin(x) = \frac{1}{2} \quad \text{or} \quad \sin(x) = 1.$$

Now solve for $x \in [0, 2\pi]$:

$$\sin(x) = 1 \Rightarrow x = \frac{\pi}{2}.$$

$$\sin(x) = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}.$$

Therefore, the solutions are:

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}.$$