

Question 10

Define the function:

$$f(x) = \frac{1 - \cos(2x)}{\sin(x)}$$

- (a) Simplify $f(x)$ using appropriate identities.
- (b) Determine the values of $x \in (0, \pi)$ for which $f(x) = 2 \sin(x)$.
- (c) State the domain of $f(x)$ on $[0, 2\pi]$.

Question 10 - Solution**(a) Simplify using identities:**

Using the identity

$$1 - \cos(2x) = 2 \sin^2(x),$$

we obtain

$$f(x) = \frac{2 \sin^2(x)}{\sin(x)}.$$

For $\sin(x) \neq 0$, this simplifies to

$$f(x) = 2 \sin(x).$$

$$f(x) = 2 \sin(x) \quad \text{for } \sin(x) \neq 0$$

(b) Solve $f(x) = 2 \sin(x)$

From part (a), the equation

$$f(x) = 2 \sin(x)$$

is an identity for all x where $f(x)$ is defined.

However, the original function is undefined when

$$\sin(x) = 0 \Rightarrow x = 0, \pi.$$

Since the interval is restricted to $(0, \pi)$, neither excluded value lies in the interval. Therefore, all values in $(0, \pi)$ satisfy the equation.

$$\text{Solution set: } (0, \pi) \quad (\text{excluded: none within the interval})$$

(c) Domain of $f(x) = \frac{1 - \cos(2x)}{\sin(x)}$ on $[0, 2\pi]$

The function is undefined when the denominator is zero:

$$\sin(x) = 0 \Rightarrow x = 0, \pi, 2\pi.$$

Removing these values from the interval $[0, 2\pi]$, the domain is

$$(0, \pi) \cup (\pi, 2\pi).$$

Excluded values: $x = 0, \pi, 2\pi$