

Question 9

Let

$$f(x) = \tan(x) - \sec(x)$$

- (a) Show that $f(x) = \frac{\sin(x)-1}{\cos(x)}$
- (b) Determine the values of $x \in (0, 2\pi)$ for which $f(x) = 0$
- (c) Analyze the behavior of $f(x)$ near $x = \frac{\pi}{2}$

Question 9 - Solution

(a) Simplify $f(x) = \tan(x) - \sec(x)$:

We write both terms in terms of sine and cosine:

$$\tan(x) = \frac{\sin(x)}{\cos(x)}, \quad \sec(x) = \frac{1}{\cos(x)}$$

Then:

$$f(x) = \frac{\sin(x)}{\cos(x)} - \frac{1}{\cos(x)} = \frac{\sin(x) - 1}{\cos(x)}$$

$$f(x) = \frac{\sin(x) - 1}{\cos(x)}$$

(b) Solve $f(x) = 0 \Rightarrow \frac{\sin(x)-1}{\cos(x)} = 0$

This occurs when the numerator is 0 (and denominator is not 0):

$$\sin(x) - 1 = 0 \Rightarrow \sin(x) = 1 \Rightarrow x = \frac{\pi}{2}$$

But at $x = \frac{\pi}{2}$, $\cos(x) = 0$, so $f(x)$ is undefined.Therefore, there is **no solution** to $f(x) = 0$ on $(0, 2\pi)$

No solution in $(0, 2\pi)$

(c) Behavior near $x = \frac{\pi}{2}$ As $x \rightarrow \frac{\pi}{2}^-$, we have: - $\sin(x) \rightarrow 1$ - $\cos(x) \rightarrow 0^+$ - So: $f(x) \rightarrow \frac{1-1}{0^+} = 0$ As $x \rightarrow \frac{\pi}{2}^+$, we have: - $\sin(x) \rightarrow 1$ - $\cos(x) \rightarrow 0^-$ - So: $f(x) \rightarrow \frac{1-1}{0^-} = 0$ But if we take x very close to $\frac{\pi}{2}$, with small deviation, say:

$$f(x) = \frac{\sin(x) - 1}{\cos(x)} \Rightarrow \text{Numerator} \approx 0, \text{ Denominator} \rightarrow 0$$

Apply L'Hôpital's Rule:

$$\lim_{x \rightarrow \frac{\pi}{2}} f(x) = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos(x)}{-\sin(x)} = \frac{0}{-1} = 0$$

So:

$$\lim_{x \rightarrow \frac{\pi}{2}} f(x) = 0$$

But since the function is undefined at $x = \frac{\pi}{2}$, the limit exists but the function is discontinuous there.