

Question 8

Let

$$f(x) = \frac{\cos(x)}{1 - \sin(x)}$$

- (a) Simplify the expression $f(x)$ using trigonometric identities.
- (b) Determine all values of $x \in (0, 2\pi)$ for which $f(x) = 1$.
- (c) Determine whether $f(x)$ is even, odd, or neither.

Question 8 - Solution

Simplification and domain. The original function is defined for $x \neq \pi/2 + 2k\pi$. Where $\cos x \neq 0$, rationalization gives

$$f(x) = \frac{1 + \sin x}{\cos x} = \sec x + \tan x.$$

At $x = 3\pi/2 + 2k\pi$, the original function has value 0, although this alternative expression is undefined.

Solve on the requested interval. If $f(x) = 1$, then $\cos x + \sin x = 1$, with $\sin x \neq 1$.

Using $\cos x = 1 - \sin x$ in $\cos^2 x + \sin^2 x = 1$ gives

$$2 \sin x (\sin x - 1) = 0.$$

The case $\sin x = 1$ is excluded. The other case requires $\sin x = 0$ and $\cos x = 1$, hence $x = 2k\pi$.

Neither endpoint belongs to $(0, 2\pi)$, so

There are no solutions in $(0, 2\pi)$.

Parity. The domain is not symmetric about zero: $-\pi/2$ is allowed but $\pi/2$ is not.

f is neither even nor odd.