

Question 7

Let

$$f(x) = \frac{1 + \cos(2x)}{\sin(2x)}$$

- (a) Simplify the expression as much as possible.
- (b) Find all $x \in (0, \pi)$ for which $f(x) = 2$.
- (c) Determine the domain of $f(x)$ over $[0, 2\pi]$.

Question 7 - Solution

Simplify. Where the original function is defined,

$$f(x) = \frac{2 \cos^2 x}{2 \sin x \cos x} = \cot x.$$

The cancellation does not restore points where $\sin(2x) = 0$.

Solve. On $(0, \pi)$, $\cot x = 2$ gives

$$x = \arctan(1/2) \approx 0.4636.$$

This point is in the original domain.

Domain. All multiples of $\pi/2$ must be excluded. On $[0, 2\pi]$ the domain is

$$(0, \pi/2) \cup (\pi/2, \pi) \cup (\pi, 3\pi/2) \cup (3\pi/2, 2\pi).$$