

Question 6

Let $f(x) = \sin(x) + \sin(3x)$

- (a) Use a trigonometric identity to write $f(x)$ as a product.
- (b) Determine all values of $x \in [0, 2\pi]$ where $f(x) = 0$.
- (c) Sketch the graph of $f(x)$ over one full period.

Question 6 - Solution

(a) Use sum-to-product identities:

We apply:

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

Let $A = 3x$, $B = x$:

$$f(x) = \sin(x) + \sin(3x) = 2 \sin \left(\frac{3x+x}{2} \right) \cos \left(\frac{3x-x}{2} \right) = 2 \sin(2x) \cos(x)$$

$$f(x) = 2 \sin(2x) \cos(x)$$

(b) Solve $f(x) = 0$ on $[0, 2\pi]$:

$$2 \sin(2x) \cos(x) = 0 \Rightarrow \sin(2x) = 0 \quad \text{or} \quad \cos(x) = 0$$

Solve $\sin(2x) = 0$:

$$2x = n\pi \Rightarrow x = \frac{n\pi}{2} \Rightarrow x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

Solve $\cos(x) = 0$:

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

Union of solutions:

$$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

(c) Sketch the graph of $f(x) = \sin(x) + \sin(3x)$ The graph is periodic with the least common multiple of the periods of $\sin(x)$ and $\sin(3x)$.Period of $\sin(x)$: 2π Period of $\sin(3x)$: $\frac{2\pi}{3}$ LCM of 2π and $\frac{2\pi}{3}$ is 2π So sketch one full period over $[0, 2\pi]$. Use the product form $f(x) = 2 \sin(2x) \cos(x)$ to help identify:Zeros: Where either $\sin(2x) = 0$ or $\cos(x) = 0$ Amplitude varies due to the product of two trig functionsImportant points: $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$, and midpoints between for peaks/troughs