

Question 5

Let $f(x) = \frac{\sin(x)}{1+\cos(x)}$

- (a) Simplify the expression $f(x)$ using trigonometric identities.
- (b) Determine all $x \in (0, 2\pi)$ for which $f(x) = 1$.
- (c) Determine the domain of $f(x)$.
- (d) Determine whether $f(x)$ is an even function, odd function, or neither.

Question 5 - Solution

Simplify with the domain retained. Half-angle identities give

$$\boxed{f(x) = \tan(x/2)}, \quad x \neq (2k+1)\pi, \quad k \in \mathbb{Z}.$$

Indeed $\sin x = 2 \sin(x/2) \cos(x/2)$ and $1 + \cos x = 2 \cos^2(x/2)$.

The alternative $\csc x - \cot x$ is valid only when $\sin x \neq 0$ and loses valid inputs such as $x = 0$.

Solve. $\tan(x/2) = 1$ implies $x = \pi/2 + 2k\pi$, so on $(0, 2\pi)$,

$$\boxed{x = \pi/2}.$$

Domain and parity. The original denominator vanishes precisely at odd multiples of π , hence

$$\boxed{D = \mathbb{R} \setminus \{(2k+1)\pi : k \in \mathbb{Z}\}}.$$

This domain is symmetric about zero, and $f(-x) = -f(x)$ on it. Therefore $\boxed{f \text{ is odd}}$.