

Question 3

Consider the function

$$f(x) = \cos(2x) - \sin(2x)$$

- (a) Write $f(x)$ as a single trigonometric function of the form $A \cos(2x + \phi)$.
- (b) Determine the amplitude and phase shift of the resulting function.
- (c) Find the maximum and minimum values of $f(x)$.
- (d) Determine all $x \in [0, 2\pi]$ such that $f(x) = 0$.

Question 3 - Solution

(a) **Write $f(x)$ as a single cosine function:**

We use the identity:

$$A \cos(2x + \phi) = A \cos \phi \cos(2x) - A \sin \phi \sin(2x)$$

Match coefficients with:

$$\cos(2x) - \sin(2x)$$

This gives the system:

$$A \cos \phi = 1, \quad A \sin \phi = 1$$

Square and add:

$$A^2(\cos^2 \phi + \sin^2 \phi) = 1^2 + 1^2 = 2 \Rightarrow A = \sqrt{2}$$

Now:

$$\cos \phi = \frac{1}{\sqrt{2}}, \quad \sin \phi = \frac{1}{\sqrt{2}} \Rightarrow \phi = \frac{\pi}{4}$$

So:

$$f(x) = \sqrt{2} \cos\left(2x + \frac{\pi}{4}\right)$$

(b) **Amplitude and Phase Shift:**

From the rewritten form:

- Amplitude: $\boxed{\sqrt{2}}$
- Phase shift: Solve $2x + \frac{\pi}{4} = 0 \Rightarrow x = -\frac{\pi}{8}$

So the graph is shifted $\boxed{\frac{\pi}{8}}$ units to the left.

(c) **Maximum and Minimum Values:**

Since cosine ranges between -1 and 1 :

$$\text{Maximum} = \sqrt{2}, \quad \text{Minimum} = -\sqrt{2}$$

(d) **Solve $f(x) = 0$ on $[0, 2\pi]$:**

$$\cos(2x) - \sin(2x) = 0 \Rightarrow \cos(2x) = \sin(2x) \Rightarrow \tan(2x) = 1$$

$$2x = \frac{\pi}{4} + n\pi \Rightarrow x = \frac{\pi}{8} + \frac{n\pi}{2}$$

Find values in $[0, 2\pi]$:

$$x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$$

Answer:

$$\boxed{x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}}$$