

Question 2

Let

$$g(x) = 2 \tan\left(x - \frac{\pi}{4}\right) + 1$$

- (a) State the period, phase shift, vertical shift, and asymptotes of $g(x)$.
- (b) Sketch one period of the graph, labeling asymptotes and intercepts.
- (c) Find the value(s) of $x \in [0, 2\pi]$ such that $g(x) = 3$.

Question 2 - Solution

(a) **Analyze** $g(x) = 2 \tan\left(x - \frac{\pi}{4}\right) + 1$

- Period: π
- Phase shift: $\frac{\pi}{4}$ to the right
- Vertical shift: $+1$

Asymptotes occur when:

$$x - \frac{\pi}{4} = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z} \Rightarrow x = \frac{3\pi}{4} + k\pi$$

So one period lies on:

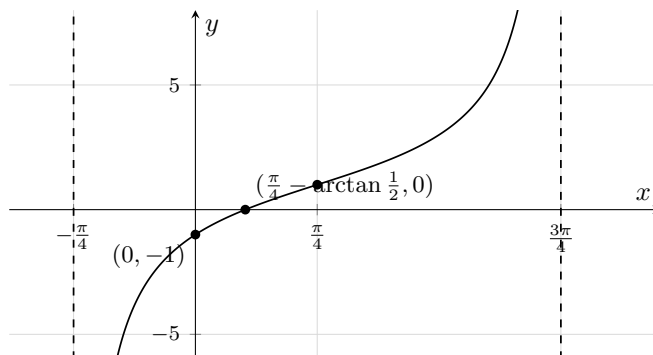
$$\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$$

- All asymptotes: $x = 3\pi/4 + k\pi, \quad k \in \mathbb{Z}$.

(b) **Sketch one period**The intercepts in this period are $(0, -1)$ and $(\pi/4 - \arctan(1/2), 0)$.

Key features:

- Left asymptote: $x = -\frac{\pi}{4}$
- Right asymptote: $x = \frac{3\pi}{4}$
- Midpoint (inflection): $x = \frac{\pi}{4}, \quad g\left(\frac{\pi}{4}\right) = 1$

(c) **Solve** $g(x) = 3$:

$$2 \tan\left(x - \frac{\pi}{4}\right) + 1 = 3 \Rightarrow \tan\left(x - \frac{\pi}{4}\right) = 1$$

$$x - \frac{\pi}{4} = \frac{\pi}{4} + n\pi \Rightarrow x = \frac{\pi}{2} + n\pi$$

Values in $[0, 2\pi]$:

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$