

Question 1

Given the function $f(x) = 3\sin\left(2x - \frac{\pi}{3}\right) + 1$, answer the following:

- (a) Determine the amplitude, period, phase shift, and vertical shift.
- (b) Find the maximum and minimum values of $f(x)$.
- (c) Sketch one full period of the graph, clearly labeling key points.
- (d) Determine the x -values where $f(x) = 1$ in the interval $[0, 2\pi]$.

Question 1 - Solution

(a) Identify parameters from $f(x) = 3 \sin(2x - \frac{\pi}{3}) + 1$:• Amplitude: $\boxed{3}$ • Period: $\frac{2\pi}{2} = \boxed{\pi}$

• Phase shift:

$$2x - \frac{\pi}{3} = 0 \Rightarrow x = \frac{\pi}{6} \Rightarrow \boxed{\frac{\pi}{6} \text{ to the right}}$$

• Vertical shift: $\boxed{+1}$

(b) Maximum and Minimum:

Since the sine function oscillates between -1 and 1 :

$$\text{Min of } f(x) = 3(-1) + 1 = -2, \quad \text{Max of } f(x) = 3(1) + 1 = 4$$

$$\boxed{\text{Max} = 4}, \quad \boxed{\text{Min} = -2}$$

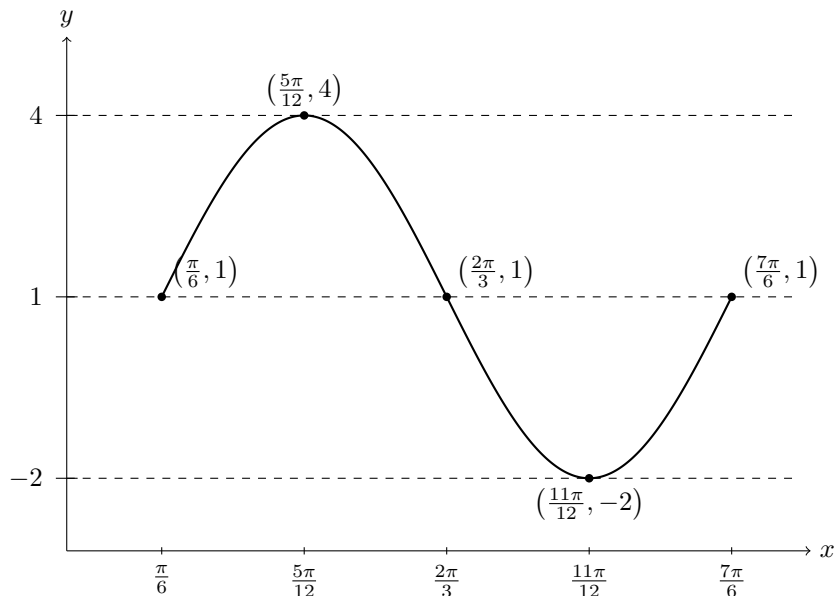
(c) Sketch one full period:

One full period has length π , starting at the phase shift:

$$x = \frac{\pi}{6} \quad \text{to} \quad x = \frac{7\pi}{6}$$

Key points over one period:

x	$f(x)$
$\frac{\pi}{6}$	1 (midline)
$\frac{5\pi}{12}$	4 (maximum)
$\frac{2\pi}{3}$	1 (midline)
$\frac{11\pi}{12}$	-2 (minimum)
$\frac{7\pi}{6}$	1 (midline)



(d) Solve $f(x) = 1$ on $[0, 2\pi]$:

$$3 \sin\left(2x - \frac{\pi}{3}\right) + 1 = 1 \Rightarrow \sin\left(2x - \frac{\pi}{3}\right) = 0$$

$$2x - \frac{\pi}{3} = n\pi \Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{6}$$

Values in $[0, 2\pi]$:

$x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$
