

Question 9

Let $f(x) = \ln(x - 2)$, with domain $x > 2$.

- (a) Find the inverse function $f^{-1}(x)$.
- (b) Determine the domain and range of f and f^{-1} .
- (c) Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

Question 9 - Solution**(a) Find the inverse:**Let $y = \ln(x - 2)$ Solve for x :

$$x - 2 = e^y \Rightarrow x = e^y + 2$$

Now switch x and y :

$$f^{-1}(x) = e^x + 2$$

Answer: $f^{-1}(x) = e^x + 2$ **(b) Domain and Range:**For $f(x) = \ln(x - 2)$: - Domain: $x > 2 \Rightarrow (2, \infty)$ - Range: $\ln(x - 2) \rightarrow -\infty$ as $x \rightarrow 2^+$, and $\rightarrow \infty$ as $x \rightarrow \infty$ Range: $(-\infty, \infty)$ For $f^{-1}(x) = e^x + 2$: - Domain: $(-\infty, \infty)$ - Range: $e^x + 2 > 2 \Rightarrow (2, \infty)$ **(c) Verify compositions:**

$$f(f^{-1}(x)) = f(e^x + 2) = \ln((e^x + 2) - 2) = \ln(e^x) = x$$

$$f^{-1}(f(x)) = f^{-1}(\ln(x - 2)) = e^{\ln(x - 2)} + 2 = x - 2 + 2 = x$$

Verified: $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$