

Question 7

Let $f(x) = \frac{6}{x+2} - 1$, with domain $x \neq -2$.

- (a) Show that f is a one-to-one function.
- (b) Find the inverse function $f^{-1}(x)$.
- (c) Determine the domain and range of both $f(x)$ and $f^{-1}(x)$.

Question 7 - Solution

(a) Show $f(x) = \frac{6}{x+2} - 1$ is one-to-one:

Let $f(x_1) = f(x_2)$. Then:

$$\frac{6}{x_1+2} - 1 = \frac{6}{x_2+2} - 1 \Rightarrow \frac{6}{x_1+2} = \frac{6}{x_2+2} \Rightarrow x_1+2 = x_2+2 \Rightarrow x_1 = x_2$$

Thus, f is one-to-one.

(b) Find the inverse function:

Let $y = \frac{6}{x+2} - 1$

Solve for x :

$$y+1 = \frac{6}{x+2} \Rightarrow x+2 = \frac{6}{y+1} \Rightarrow x = \frac{6}{y+1} - 2$$

Now switch x and y :

$$f^{-1}(x) = \frac{6}{x+1} - 2$$

Answer: $f^{-1}(x) = \frac{6}{x+1} - 2$

(c) Domain and Range:

Original function $f(x) = \frac{6}{x+2} - 1$

- Undefined when $x+2=0 \Rightarrow x=-2$ - Domain of f : $(-\infty, -2) \cup (-2, \infty)$

Asymptote at $x=-2$, and horizontal asymptote $y=-1$ - Range of f : $(-\infty, -1) \cup (-1, \infty)$

Inverse: - Domain of f^{-1} : same as range of f : $(-\infty, -1) \cup (-1, \infty)$ - Range of f^{-1} : same as domain of f : $(-\infty, -2) \cup (-2, \infty)$