

Question 10

Sketch the graph of the function:

$$f(x) = \frac{x}{\sqrt{x^2 + 1}}$$

Instructions: Identify the domain, range, intercepts, asymptotic behavior, and any symmetry. Then sketch the graph clearly.

Question 10 - Solution

We are given:

$$f(x) = \frac{x}{\sqrt{x^2 + 1}}$$

Step 1: Domain

The denominator is always positive and defined for all real x :

$$\text{Domain: } (-\infty, \infty)$$

Step 2: Intercepts

Y-intercept:

$$f(0) = \frac{0}{\sqrt{0^2 + 1}} = 0$$

X-intercept:

$$f(x) = 0 \Rightarrow \frac{x}{\sqrt{x^2 + 1}} = 0 \Rightarrow x = 0$$

Step 3: Symmetry

Check if the function is odd:

$$f(-x) = \frac{-x}{\sqrt{(-x)^2 + 1}} = \frac{-x}{\sqrt{x^2 + 1}} = -f(x)$$

So the function is **odd**, symmetric about the origin.

Step 4: End Behavior

As $x \rightarrow \pm\infty$:

$$f(x) = \frac{x}{\sqrt{x^2 + 1}} \rightarrow \pm 1$$

So horizontal asymptotes:

$$\boxed{y = 1} \quad \text{as } x \rightarrow \infty \quad \text{and} \quad \boxed{y = -1} \quad \text{as } x \rightarrow -\infty$$

Step 5: Range

$$\text{Range: } (-1, 1)$$

The function approaches 1 and -1 but never reaches them.

