

Question 9

Sketch the graph of the function:

$$f(x) = \ln(x^2 + 1)$$

Instructions: Identify the domain, intercepts, end behavior, and any symmetry. Then sketch the graph.

Question 9 - Solution

We are given:

$$f(x) = \ln(x^2 + 1)$$

Domain: Since $x^2 + 1 > 0$ for all real x ,

Domain: $(-\infty, \infty)$

Y-intercept:

$$f(0) = \ln(0^2 + 1) = \ln(1) = 0$$

X-intercepts: Set $f(x) = 0$:

$$\ln(x^2 + 1) = 0 \Rightarrow x^2 + 1 = 1 \Rightarrow x^2 = 0 \Rightarrow x = 0$$

So the only intercept is at the origin.

Symmetry:

$$f(-x) = \ln((-x)^2 + 1) = \ln(x^2 + 1) = f(x) \Rightarrow \text{even function}$$

End behavior: As $x \rightarrow \pm\infty$, $x^2 + 1 \rightarrow \infty \Rightarrow f(x) \rightarrow \infty$

Range: Minimum value is at $x = 0$: $\ln(1) = 0$, and it increases without bound.

Range: $[0, \infty)$

