

Question 7

Sketch the graph of the function:

$$f(x) = \frac{1}{x^2 + 1}$$

Instructions: Identify the domain, range, intercepts, end behavior, and any symmetry of the function. Then sketch the graph.

Question 7 - Solution

We are given:

$$f(x) = \frac{1}{x^2 + 1}$$

Step 1: Domain

Since $x^2 + 1 > 0$ for all real x , the domain is:

$$(-\infty, \infty)$$

Step 2: Y-intercept

Set $x = 0$:

$$f(0) = \frac{1}{0^2 + 1} = 1$$

Step 3: X-intercepts

Set $f(x) = 0$:

$$\frac{1}{x^2 + 1} = 0 \Rightarrow \text{no solution}$$

So, **no x-intercepts**.

Step 4: Range

Since the denominator is always ≥ 1 , the maximum occurs at $x = 0$, and the function decreases on both sides. Therefore:

$$\text{Range: } (0, 1]$$

Step 5: Symmetry

$$f(-x) = \frac{1}{(-x)^2 + 1} = \frac{1}{x^2 + 1} = f(x)$$

So the function is **even**, symmetric about the y-axis.

Step 6: End Behavior

As $x \rightarrow \pm\infty$, $f(x) \rightarrow 0$

