

Question 6

Sketch the graph of the rational function:

$$f(x) = \frac{x^2 - 1}{x^2 - 4}$$

Instructions: Identify all intercepts, vertical and horizontal asymptotes, domain, and any removable discontinuities. Then sketch the graph based on your analysis.

Question 6 - Solution

We are given:

$$f(x) = \frac{x^2 - 1}{x^2 - 4} = \frac{(x-1)(x+1)}{(x-2)(x+2)}$$

Domain: $x \neq \pm 2 \Rightarrow (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

Vertical Asymptotes: $x = -2$ and $x = 2$

Horizontal Asymptote: Degrees of numerator and denominator are equal: $y = \frac{1}{1} = y = 1$

X-intercepts: Set numerator = 0: $x = \pm 1$

Y-intercept: $f(0) = \frac{-1}{-4} = \frac{1}{4}$

