

Question 3

Sketch the graph of the piecewise function:

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 0 \\ 2x + 1 & \text{if } x \geq 0 \end{cases}$$

Instructions: Clearly identify continuity, domain, range, intercepts, and sketch the graph.

Question 3 - Solution

We are given a piecewise function:

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 0 \\ 2x + 1 & \text{if } x \geq 0 \end{cases}$$

Step 1: Domain

Both pieces are defined on their respective intervals, so:

$$\text{Domain: } (-\infty, \infty)$$

Step 2: Range

For $x < 0$: $f(x) = x^2 + 1 \Rightarrow f(x) > 1$, For $x \geq 0$: $f(x) = 2x + 1 \Rightarrow f(x) \geq 1$

So the function reaches its minimum at $f(0) = 1$, and increases in both directions:

$$\text{Range: } [1, \infty)$$

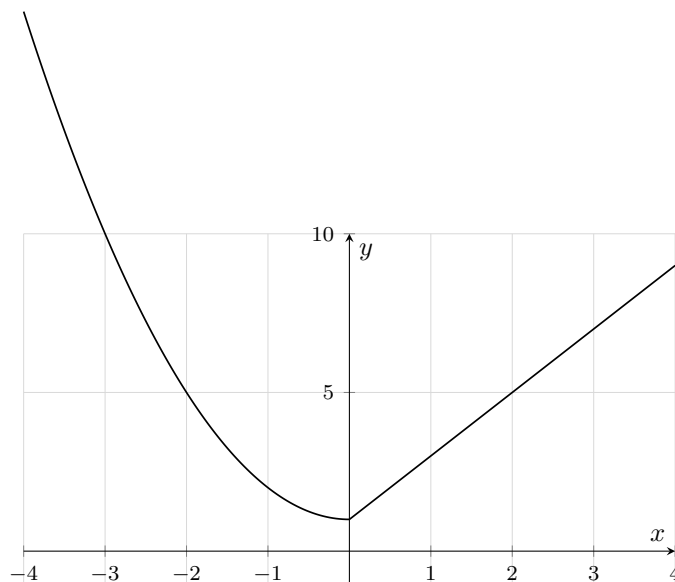
Step 3: Continuity at $x = 0$

$$\lim_{x \rightarrow 0^-} f(x) = 0^2 + 1 = 1, \quad \lim_{x \rightarrow 0^+} f(x) = 2(0) + 1 = 1 \Rightarrow \text{Function is continuous at } x = 0$$

Step 4: Intercepts

Y-intercept: $f(0) = 1$ **X-intercept:** Solve $f(x) = 0$: , $x^2 + 1 = 0 \Rightarrow$ no real solution , $2x + 1 = 0 \Rightarrow x = -\frac{1}{2} \notin [0, \infty) \Rightarrow$ no x-intercept

Conclusion: Domain: $(-\infty, \infty)$, Range: $[1, \infty)$, No x-intercepts, y-intercept at $(0, 1)$, Continuous everywhere



$$f(x) = \begin{cases} x^2 + 1 & x < 0 \\ 2x + 1 & x \geq 0 \end{cases}$$