

Question 10

Let the function f be defined by:

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2 \\ c, & x = 2 \end{cases}$$

- (a) Find the limit $\lim_{x \rightarrow 2} f(x)$.
- (b) Determine the value of c that makes $f(x)$ continuous at $x = 2$.
- (c) Is $f(x)$ differentiable at $x = 2$ for the value of c you found? Justify your answer.

Question 10 - Solution

(a) Compute $\lim_{x \rightarrow 2} f(x)$:

Factor the numerator:

$$f(x) = \frac{x^2 - 4}{x - 2} = \frac{(x - 2)(x + 2)}{x - 2}, \quad x \neq 2$$

Cancel the common factor:

$$f(x) = x + 2 \quad \text{for } x \neq 2$$

So:

$$\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} (x + 2) = 4$$

Answer: $\boxed{4}$

(b) To make f continuous at $x = 2$, set:

$$f(2) = \lim_{x \rightarrow 2} f(x) = 4 \Rightarrow c = \boxed{4}$$

(c) **Differentiability at $x = 2$:**

With $c = 4$, we now have:

$$f(x) = \begin{cases} x + 2, & x \neq 2 \\ 4, & x = 2 \end{cases}$$

The function is: Continuous at $x = 2$, Equal to a linear function near $x = 2$

Compute derivative using definition:

$$f'(2) = \lim_{h \rightarrow 0} \frac{f(2 + h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{(2 + h + 2) - 4}{h} = \frac{4 + h - 4}{h} = \lim_{h \rightarrow 0} \frac{h}{h} = 1$$

Conclusion: $f(x)$ is differentiable at $x = 2$, with $f'(2) = \boxed{1}$