

Question 9

Let $f(x) = \begin{cases} 2x + 5 & \text{if } x < 1 \\ x^2 + 2 & \text{if } x \geq 1 \end{cases}$

- (a) Is $f(x)$ continuous at $x = 1$? Justify analytically.
- (b) Is $f(x)$ differentiable at $x = 1$? If not, explain why.

Question 9 - Solution**(a) Continuity at $x = 1$:**

Check:

$$\lim_{x \rightarrow 1^-} f(x) = 2(1) + 5 = 7$$

$$\lim_{x \rightarrow 1^+} f(x) = 1^2 + 2 = 3$$

Since:

$$\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x),$$

The two-sided limit does not exist, so:

$f(x)$ is not continuous at $x = 1$

(b) Differentiability at $x = 1$:

A function cannot be differentiable at a point where it is not continuous.

So:

$f(x)$ is not differentiable at $x = 1$

Explanation: The left and right-hand limits do not match at $x = 1$, so there is a jump discontinuity.