

Question 8

Let

$$f(x) = \frac{1}{x}, \quad g(x) = \sqrt{x-2}.$$

- (a) Find the composite function $(f \circ g)(x)$ and state its domain.
- (b) Determine whether $(f \circ g)(x)$ is defined at $x = 2$. Justify your answer.
- (c) Evaluate $(f \circ g)(6)$.

Question 8 - Solution**(a) Composite function and domain**

The composite function is

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x-2}) = \frac{1}{\sqrt{x-2}}.$$

To determine the domain:

- The square root requires $x - 2 \geq 0 \Rightarrow x \geq 2$.
- The denominator cannot be zero, so $\sqrt{x-2} \neq 0 \Rightarrow x \neq 2$.

Thus, the domain is

$$\boxed{(2, \infty)}.$$

(b) Endpoint analysis

At $x = 2$,

$$(f \circ g)(2) = \frac{1}{\sqrt{2-2}} = \frac{1}{0},$$

which is undefined. Therefore, $x = 2$ is excluded from the domain.

(c) Evaluation

Evaluate the composite at $x = 6$:

$$(f \circ g)(6) = \frac{1}{\sqrt{6-2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}.$$

Answer:

$$\boxed{\frac{1}{2}}$$