

Question 7

Consider the function

$$f(x) = \frac{x^2 + 2x - 3}{x^2 - 4}.$$

- (a) Determine all vertical and horizontal asymptotes of $f(x)$.
- (b) Identify any holes in the graph of $f(x)$, and specify their coordinates.
- (c) State the domain of $f(x)$.

Question 7 - Solution

We are given

$$f(x) = \frac{x^2 + 2x - 3}{x^2 - 4}.$$

Factor the numerator and denominator:

$$f(x) = \frac{(x+3)(x-1)}{(x-2)(x+2)}.$$

(a) Vertical asymptotes occur where the denominator is zero and no factor cancels. Since

$$x^2 - 4 = (x-2)(x+2),$$

the denominator is zero at

$$x = 2 \quad \text{and} \quad x = -2.$$

There are no common factors with the numerator, so both are vertical asymptotes:

$$x = -2, \quad x = 2.$$

Since the numerator and denominator have the same degree, the horizontal asymptote is the ratio of leading coefficients:

$$y = \frac{1}{1} = 1.$$

Thus,

$$y = 1.$$

(b) Holes occur when a factor cancels from the numerator and denominator. Since

$$\frac{(x+3)(x-1)}{(x-2)(x+2)}$$

has no common factors, there are

$$\text{no holes}.$$

(c) The domain excludes the values that make the denominator zero:

$$x \neq -2, \quad x \neq 2.$$

Therefore,

$$(-\infty, -2) \cup (-2, 2) \cup (2, \infty).$$

